

Physics of Zonal Flows

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This overview is dedicated to the memory of
Professor Marshall N. Rosenbluth.



Contents

What is a zonal flow?

Basic Physics, Impact on Transport, ZF vs. mean $\langle E_r \rangle$, Self-regulation

Why ZF is Important for Fusion ?

Assessment: "*What we understood*"

Universality, Damping and Growth, Unifying Concept: Self-regulating dynamics

Current Research: "*What we think we understand*"

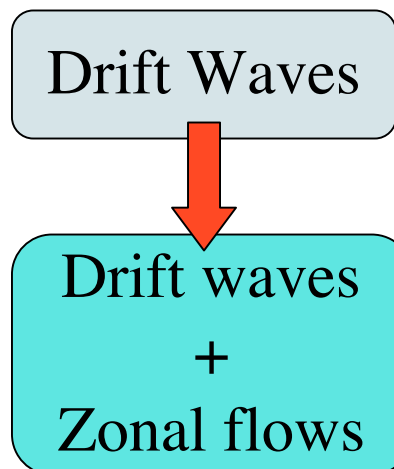
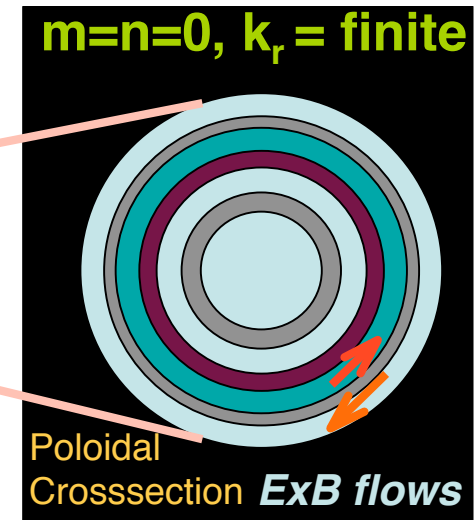
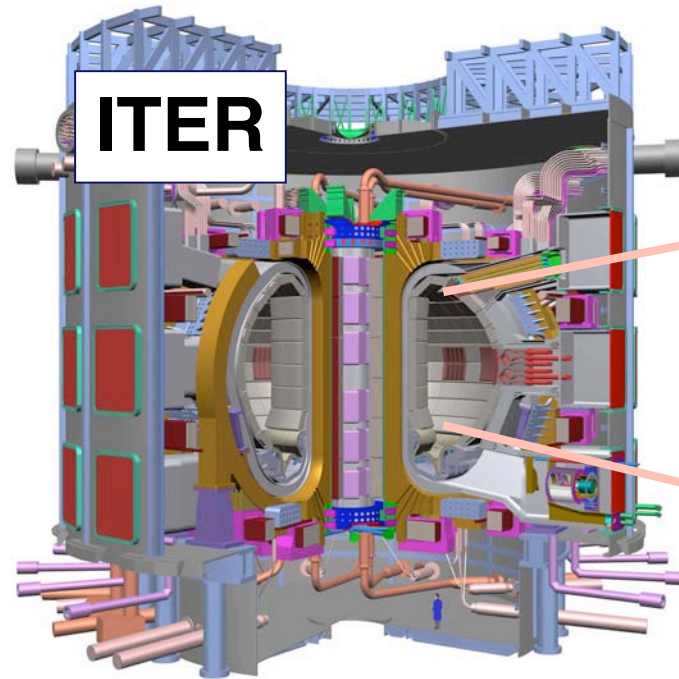
Existence, Collisionless Saturation, Marginality, ZF and $\langle E_r \rangle$, Control Knobs

Future Tasks: "*What we do not understand*"

Summary

Acknowledgements

What is a zonal flow?

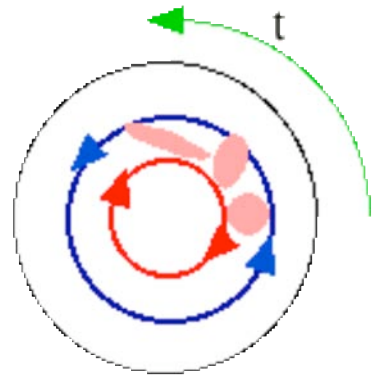


Paradigm
Change

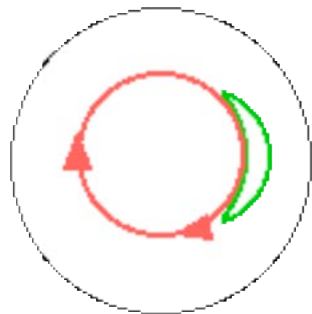
ZFs are "mode", but:

1. Turbulence driven
2. No linear instability
3. No direct radial transport

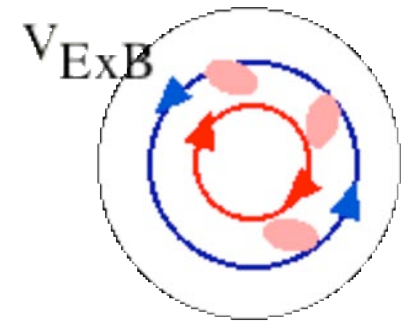
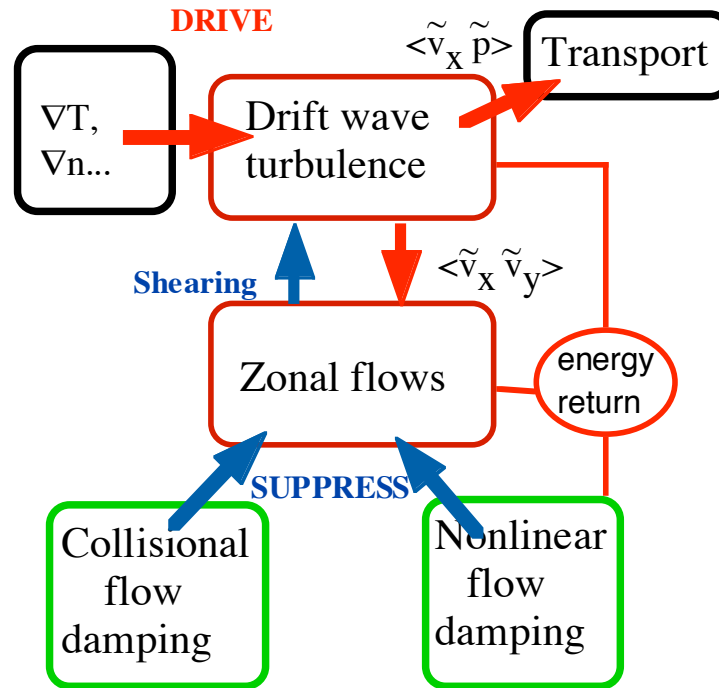
Basic Physics of a zonal flow



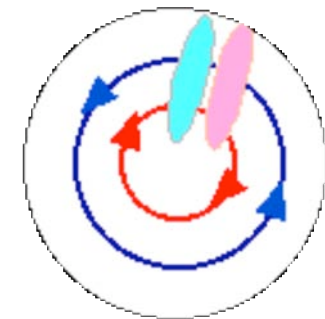
Suppression of DW by shearing



Damping by collisions



Generation By vortex tilting



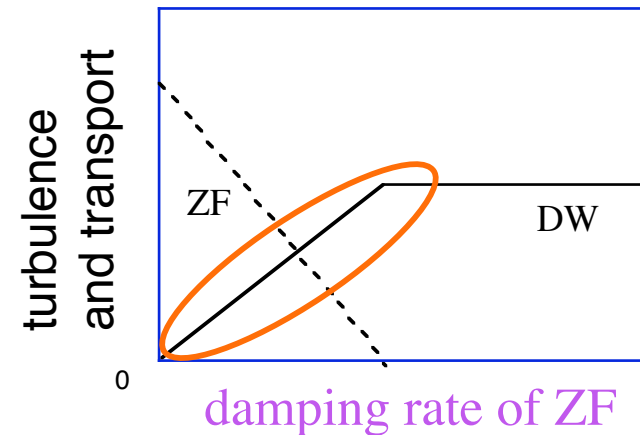
Tertiary instability

Self-regulation: Co-existence of ZF and DW

$$\left\{ \begin{array}{l} \frac{\partial}{\partial t} W_d = \gamma [\nabla p_0, \dots] W_d - \alpha W_d W_{ZF} \\ \frac{\partial}{\partial t} W_{ZF} = \gamma_{\text{damp}} [\dots] W_{ZF} + \alpha W_d W_{ZF} \end{array} \right. \quad \begin{array}{l} W_d : \text{drift wave energy} \\ W_{ZF} : \text{zonal flow energy} \end{array}$$

↓ $\left(\begin{array}{l} v_{ii}, q, \varepsilon \\ \text{geometry} \end{array} \right) \quad \left(+ \text{rf, etc.} \right)$

$$W_d \sim \frac{\gamma_{\text{damp}}}{\alpha}$$



Transport
coefficient



$$\chi_i \sim \frac{\gamma_{\text{damp}}}{\omega_{\text{eff}}} \chi_{gB} \quad \rightarrow \quad \chi_i = \mathcal{R} \chi_{gB}$$

"R-Factor"

Confinement enhancement
Includes other reduction effects
(i.e., cross phase)

Why ZFs are Important for Fusion ?

Issues in fusion

1. Self-regulation

$$\chi = \mathcal{R} \chi_{gB}$$

2. Shift for onset of large transport

3. Identify transport control knob

4. Meso-scales and zonal field

$$\tau_E = H \tau_{E,L} - \text{scaling}$$

"Intrinsic" H-factor (wo barrier)

$$H \propto \mathcal{R}^{-0.6}$$

Operation of ITER
near marginality

Flow damping ?

Intermittent flux,
e.g., ELMs

Impacts on
RWM and NTM

Realizability of
Ignition

$$\$ \propto \mathcal{R}^{-0.8}$$

$$\$ \propto H^{-1.3}$$

P_{LH} - threshold

Peak heat load

β -limit

Steady state

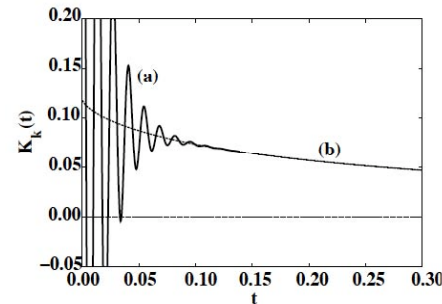
Assessment I "What we understood"

ZFs are UNIVERSAL

	Scales	Transport	Role of ZF	This IAEA Conference
ITG	ρ_i c_s/a	core χ_i, χ_ϕ, D	Very important	Falchetto (TH/1-3Rd), Hahm (TH/1-4), Hallatschek (TH/P6-3), Hamaguchi (TH/8-3Ra), Miyato (TH/8-5Ra), Waltz (TH/8-2), Watanabe (TH/8-3Rb)
TEM/TIM	ρ_i c_s/a	core $\chi_i, \chi_e, \chi_\phi, D$	Very important	Lin (TH/8-4), Sarazin (TH/P6-7), Terry (TH/P6-9)
ETG	ρ_e $v_{Th,e}/a$	core χ_e, χ_J	On-going	Holland (TH/P6-5), Horton (TH/P3-5), Idomura (TH/8-1), Li (TH/8-5Ra), Lin (TH/8-4)
Resistive ballooning/ interchange	$\Delta_{\text{resis. layer}}$	edge, sol, helical edge	Important	Benkadda(TH/1-3Rb), del-Castillo-Negrette (TH/1-2)
Drift Alfvén waves at edge	$\sim \rho_i$	edge, sol	Very important	Scott(TH/7-1), Shurygin(TH/P6-8)

Linear Damping of Zonal Flows

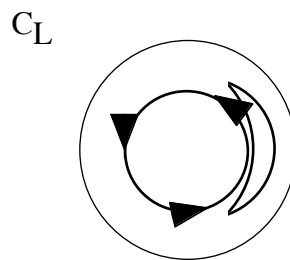
Neoclassical process



Role of bananas



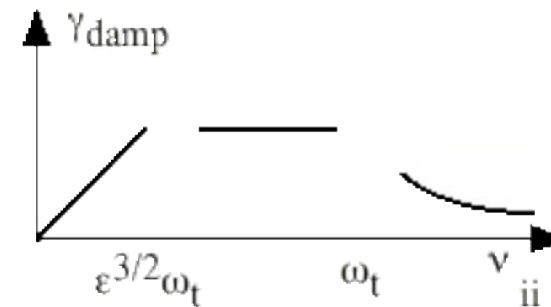
Frictional damping



Rosenbluth-Hinton
undamped flow - survives for
 $\tau > \tau_{\text{transport}}$

$$\frac{\phi_q(t)}{\phi_q(0)} = \frac{1}{1 + 1.6\epsilon^{-1/2}q^2}$$

Banana-plateau transition

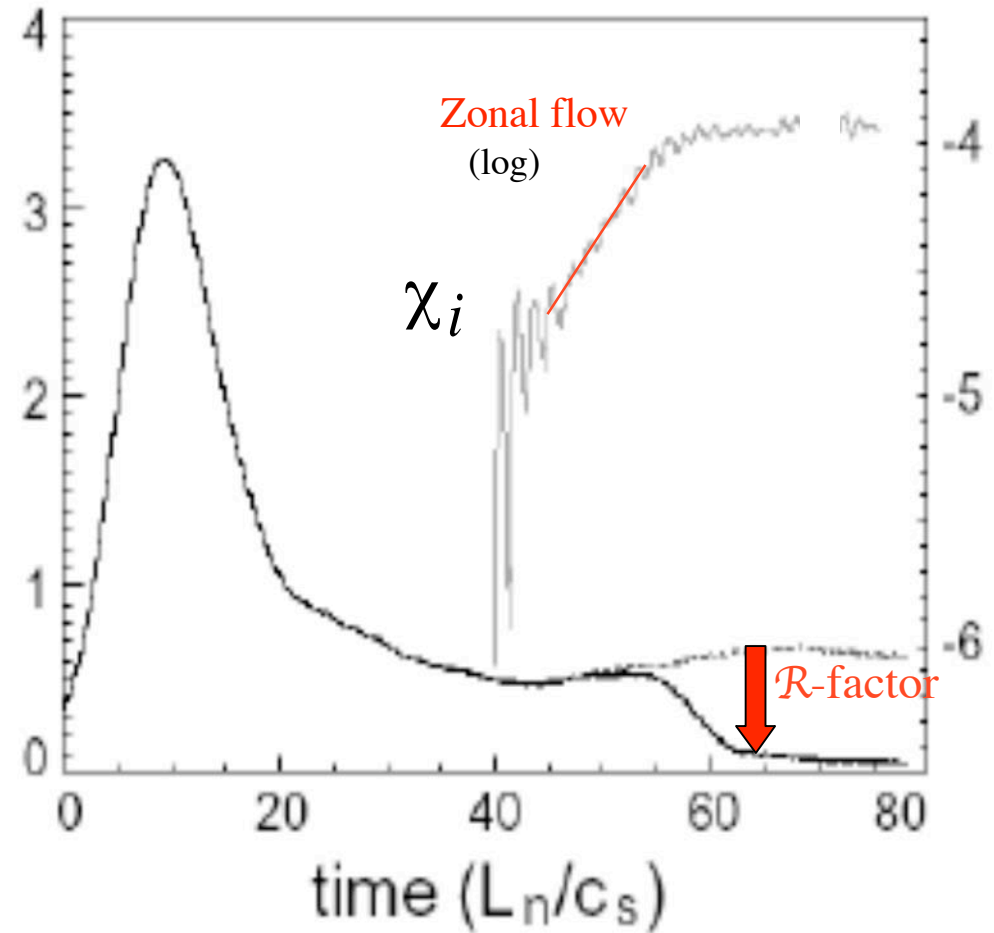
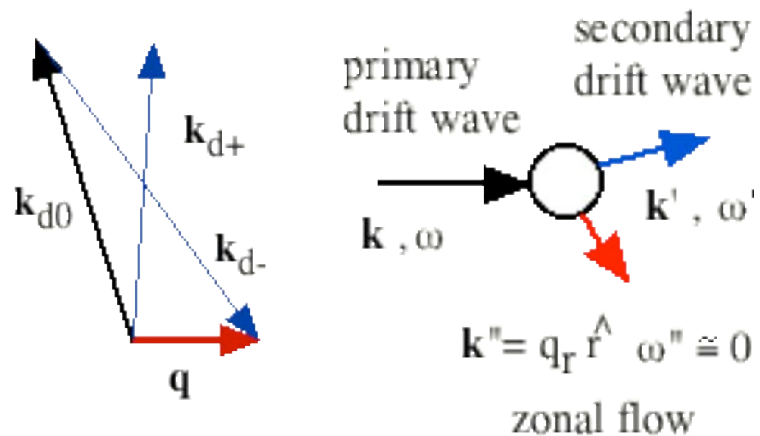


$$\gamma_{\text{damp}} \simeq \nu_{ii}/\epsilon \Rightarrow \chi_i \propto \nu_{ii} \quad \text{even in "collisionless" regime}$$

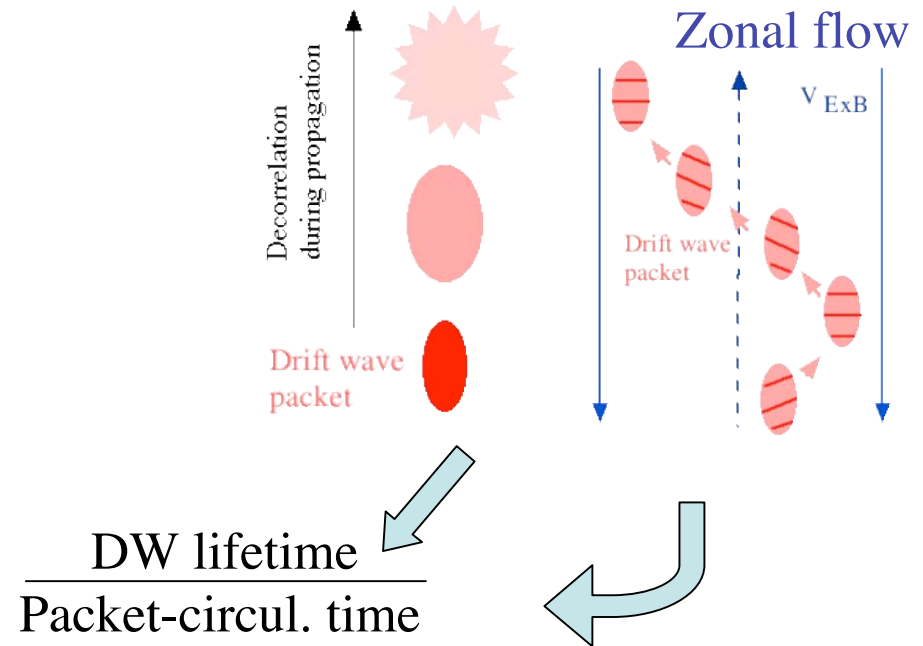
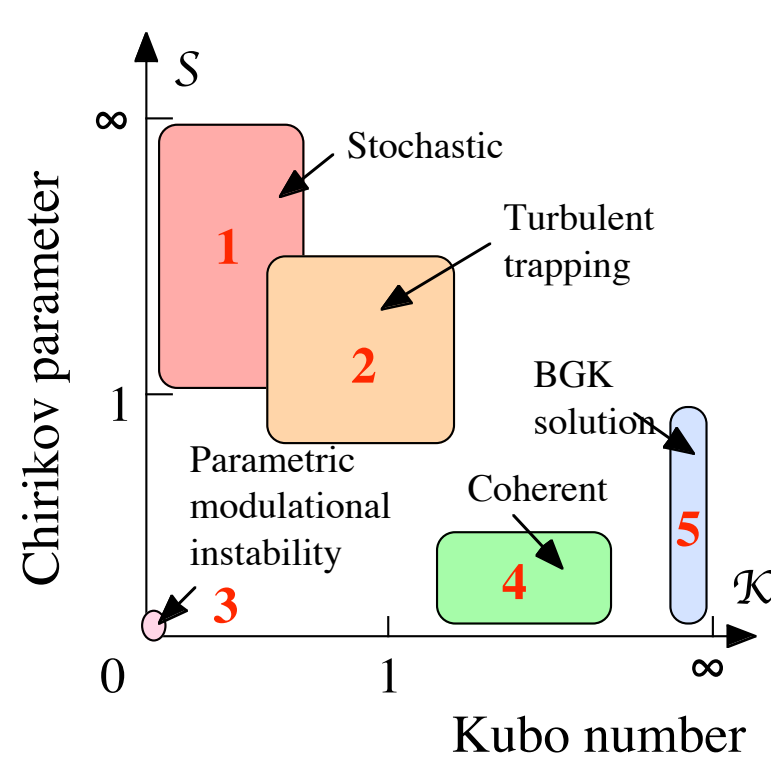
Screening effect if $q_r \rho_p \sim O(1)$

Growth Mechanism

ZFs by
modulational instability



Numerical experiment indicates instability of finite amplitude gas of drift waves to zonal shears



Regime

Keywords

- 1 k_r Diffusion
- 2 Turbulent trapping
- 3 Single wave modulation
- 4 Reductive perturbation
- 5 DW trapping in ZF

References

Zakharov, PHD, Itoh, Kim, Krommes
 Balescu
 Sagdeev, Hasegawa, Chen, Zonca
 Taniuti, Weiland, Champeaux
 Kaw

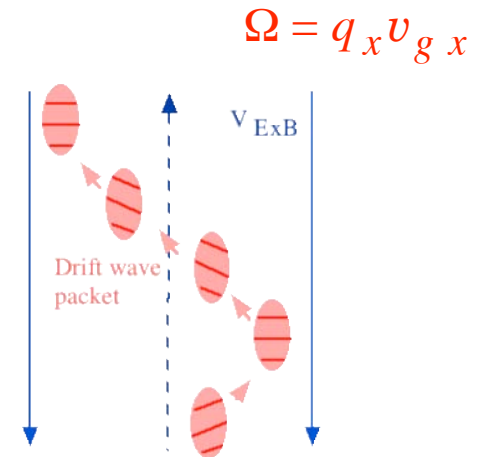
Close Relationship: DW + ZF and Vlasov Plasma

(i) DW + ZF :
$$\frac{dx}{dt} = v_{gx}(k) \quad \frac{dk_x}{dt} = -\frac{\partial}{\partial x}(k_y \tilde{V}_Z)$$

$$\frac{dy}{dt} = v_{gy}(k) \quad \frac{dk_y}{dt} = 0$$
 \Rightarrow 'Ray' Trapping

$$\frac{\partial}{\partial t} \tilde{V}_Z + \gamma_{\text{damp}} \tilde{V}_Z = -\frac{\partial}{\partial x} \langle \tilde{V}_x \tilde{V}_y \rangle$$

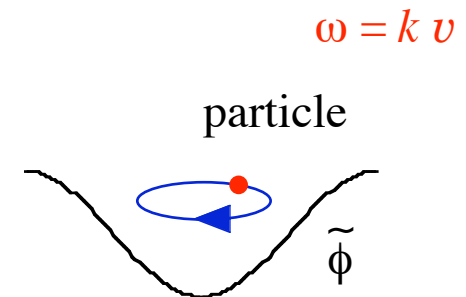
$$\frac{\partial}{\partial t} N + v_{gx} N - \frac{\partial}{\partial x} (k_y V_Z) \frac{\partial N}{\partial k_x} = \gamma_k N + C_k(N)$$



(ii) 1D Vlasov Plasma :
$$\frac{dx}{dt} = v \quad \frac{dv}{dt} = \frac{e}{m} \tilde{E}$$
 \Rightarrow Particle Trapping

$$\frac{\partial \tilde{E}}{\partial x} = 4\pi n_0 e \int dv f$$

$$\frac{\partial f}{\partial t} + v \frac{df}{dx} + \frac{e}{m} \tilde{E} \frac{\partial f}{\partial v} = C(f)$$



Note: Conservation energy between ZF and DW

RPA equations

$$\begin{aligned}
 \text{DW} \quad \frac{\partial}{\partial t} |\tilde{V}_{\text{DW}}|^2 + \sum_{\mathbf{k}} (\gamma_{L, \mathbf{k}} + C_{\mathbf{k}}(N)) |\tilde{V}_{\text{DW}, \mathbf{k}}|^2 &= \frac{2}{B^2} \sum_{q_x} \int d^2k \frac{q_x^2 k_{\perp}^2 k_x |V_{\text{ZF}, q}|^2}{(1 + k_{\perp}^2 \rho_s^2)^2} R(k, q_x) \frac{\partial \langle N \rangle}{\partial k_x} \\
 \text{ZF} \quad \left(\frac{\partial}{\partial t} + \gamma_{\text{damp}} \right) |V_{\text{ZF}}|^2 &= - \frac{2}{B^2} \sum_{q_x} \int d^2k \frac{q_x^2 k_{\perp}^2 k_x |V_{\text{ZF}, q}|^2}{(1 + k_{\perp}^2 \rho_s^2)^2} R(k, q_x) \frac{\partial \langle N \rangle}{\partial k_x}
 \end{aligned}$$

Coherent equations

$$\begin{aligned}
 \text{DW} \quad \frac{dP^2}{d\tau} &= P^2 - 2 P Z S \cos(\Psi) \quad (\text{S: beat wave}) \\
 \text{ZF} \quad \frac{dZ^2}{d\tau} &= - \frac{\gamma_{\text{damp}}}{\gamma_L} Z^2 + 2 P Z S \cos(\Psi)
 \end{aligned}$$

$$\left. \frac{\partial}{\partial t} W_{\text{d}} \right|_{\text{by ZF}} = - \left. \frac{\partial}{\partial t} W_{\text{ZF}} \right|_{\text{by DW}}$$

Self-regulating System Dynamics

Simplified Predator-Prey model

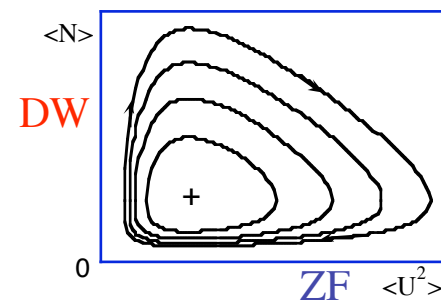
$$\text{DW} \quad \frac{\partial}{\partial t} \langle N \rangle = \gamma_L \langle N \rangle - \gamma_2 \langle N \rangle^2 - \alpha \langle U^2 \rangle \langle N \rangle$$

$$\text{ZF} \quad \frac{\partial}{\partial t} \langle U^2 \rangle = -\gamma_{\text{damp}} \langle U^2 \rangle + \alpha \langle U^2 \rangle \langle N \rangle$$

Cyclic bursts

$$\gamma_2 \rightarrow 0$$

(No self damping)

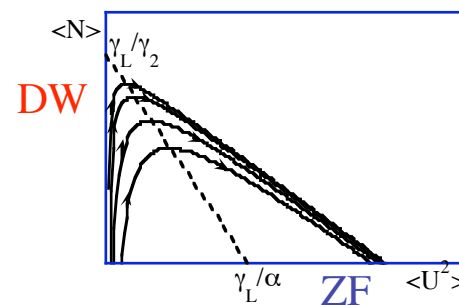


Single burst

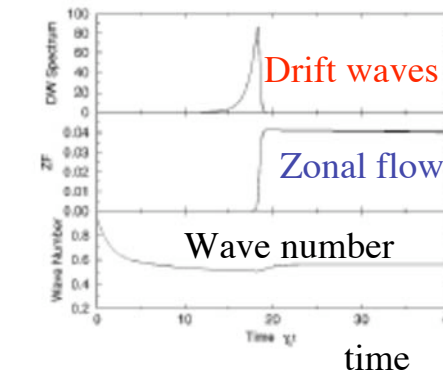
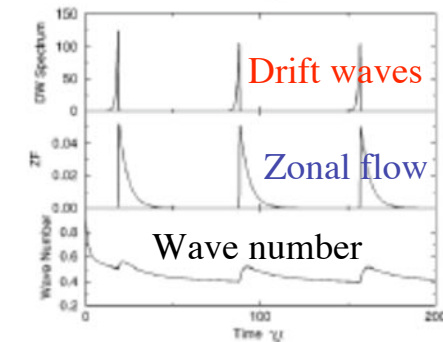
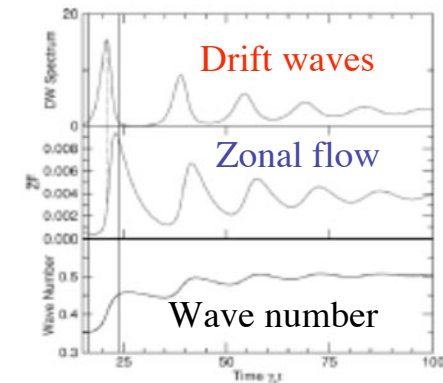
(Dimits shift)

$$\gamma_{\text{damp}} \rightarrow 0$$

(No ZF friction)



Stable fixed point

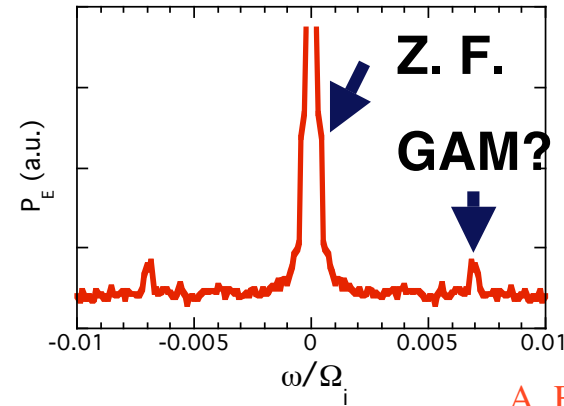
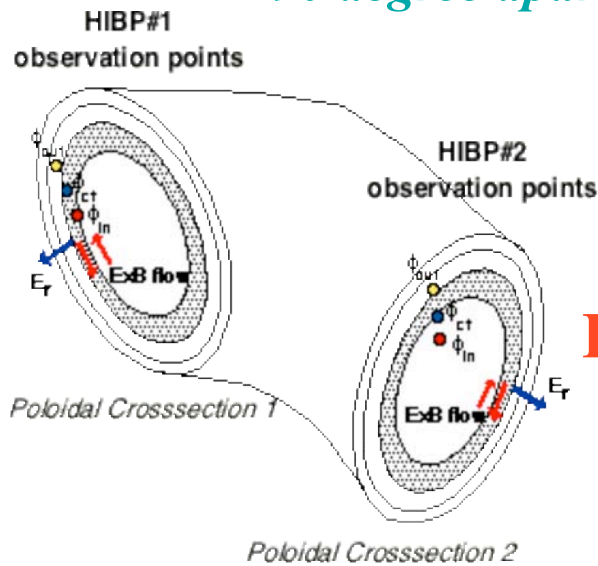


II Current research: "What we think we understand"¹⁴

Zonal flows really do exist

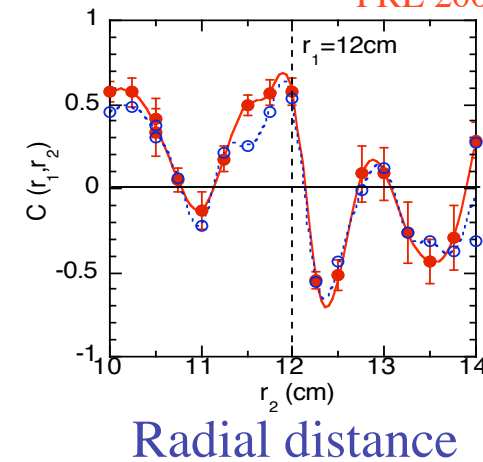
CHS Dual HIBP System

90 degree apart



A. Fujisawa et al.,
PRL 2004 in press

$E_r(r,t)$



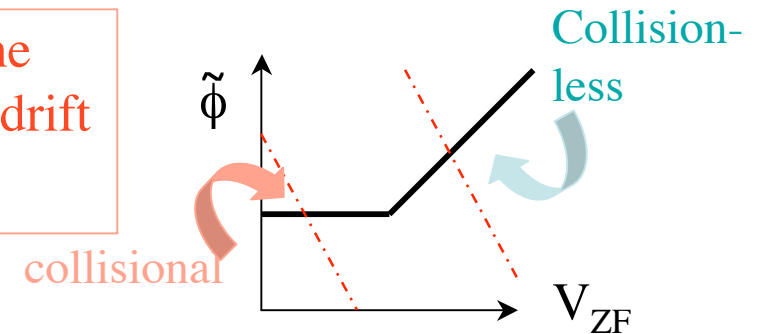
$E_r(r,t)$ { High correlation on magnetic surface,
Slowly evolving in time,
Rapidly changing in radius.



Candidates for Collisionless Saturation

$$\chi = \frac{\gamma_{nl}^{\text{collisionless}}}{\omega_{\text{eff}}} \chi_{gB}$$

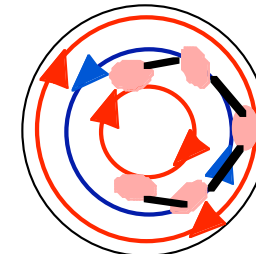
Partial recovery of the dependence of χ_i on drift wave growth rate



Trapping

Plateau in
k-space $N(k)$

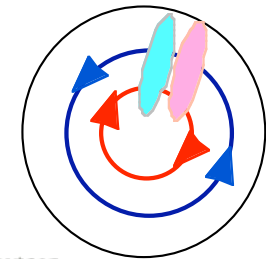
$$\sqrt{V_{ZF}} \propto \omega_{\text{bounce}} \approx \Delta\omega, \gamma_L$$



Tertiary Instability

Returns energy
to drift waves

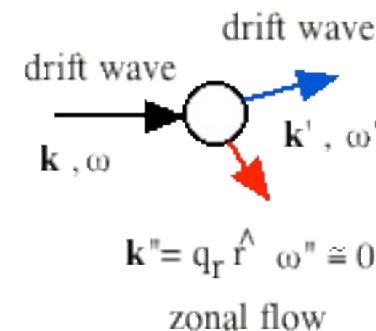
$$V_{ZF} \sim \left| \hat{\phi} + \frac{T_e}{2T_i} \hat{T} \right| q_r$$



Higher Order Kinetics

Analogous to
interaction at beat
wave resonance

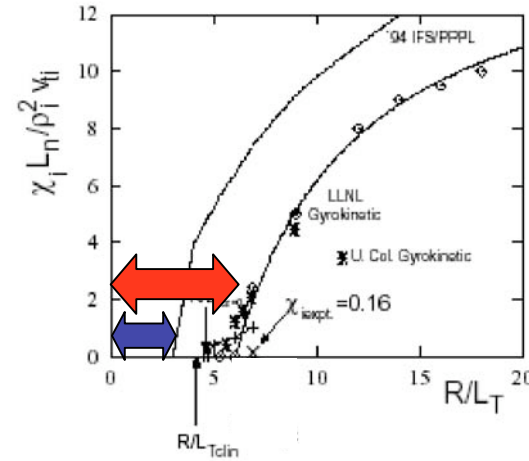
$$V_{ZF} \sim \Delta\omega k_{\theta}^{-1}$$



"Near" Marginality

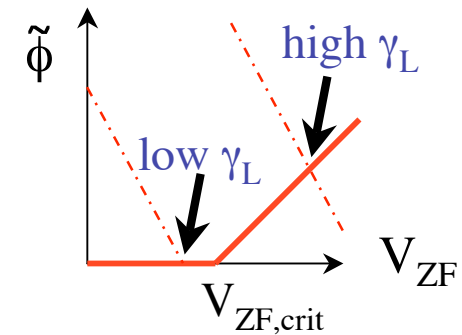
Shift exists

ITER plasma near marginality \Rightarrow
Importance of ZF



Where the energy goes

**What limits the "nearly marginal" region:
(Dimits shift)**



Mechanisms: Trapping, Tertiary, Higher order nonlinearity,....

An example: $\gamma_{L, \text{crit}} \sim q_r^2 k_{\theta}^{-2} \alpha$

(Higher-order nonlinearity model)

Route to the shift is understood, but "the number" is not yet obtained.

Electromagnetic Effect

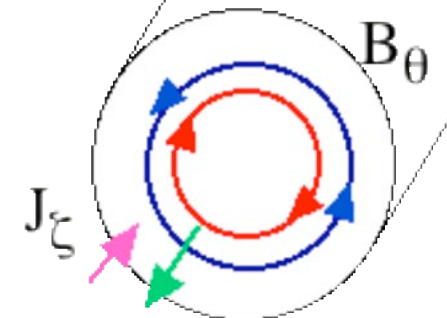
Subject	Mechanism for ZF growth	Implications for fusion
ZF generation by finite- β drift waves	Modulational instability of a drift Alfvén wave	Transport at high- β , L-H transition
Zonal magnetic field generation	Random refraction of Alfvén wave turbulence	Possible Z-field induced transition, NTM, ...

$$\langle \tilde{v}_r \tilde{v}_\theta \rangle \Rightarrow \langle \tilde{v}_r \tilde{v}_\theta \rangle - \langle \tilde{B}_r \tilde{B}_\theta \rangle$$

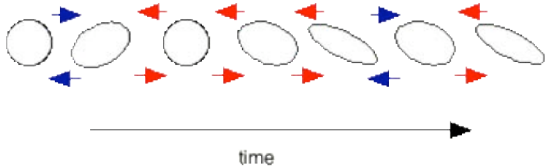
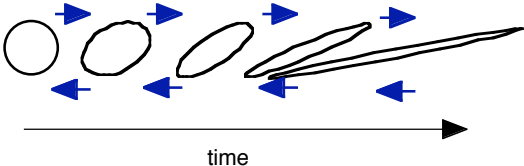
Selected structure
 Zonal flow \Rightarrow Zonal field
 $\beta \longrightarrow$

$$\frac{\partial}{\partial t} B_\theta = -\eta_{ZF} \nabla^2 B_\theta \quad \eta_{ZF} = -\frac{4\pi c_s^2 \delta_e^2}{v_{th,e} (1 + q_r^2 \delta_e^2)} \sum_k \frac{(1 + k_\perp^2 \rho_s^2)^{5/2}}{2 + k_\perp^2 \rho_s^2} \frac{k_\perp^2 k_y^2}{|k_\parallel|} \frac{\partial^2}{\partial k_x^2} \left(\frac{\langle \omega_k N_k \rangle}{\sqrt{1 + k_\perp^2 \rho_s^2}} \right) f_0$$

- \Rightarrow Current layer generation
- Corrugated magnetic shear,
- Tertiary micro tearing mode ?
- \Rightarrow Can seed Neoclassical Tearing Mode
- Localized current at separatrix of tearing mode,



Distinction between ZF and mean Field $\langle E_r \rangle$

	Zonal Flows	Mean Field $\langle E_r \rangle$
Time	can change on turbulence time scales	changes on transport time scales
Space	oscillating, complex pattern in radius $\sim 20 \rho_i$	smoothly varying
Stretching Behavior k of waves	diffusive $\langle \delta k^2 \rangle \propto t$ 	ballistic $\langle \delta k^2 \rangle = t^2 k^2 V_E'^2$ 
Drive	Turbulence	equilibrium ∇p , orbit loss, external torque, turbulence, etc.

Interplay of Zonal Flow and Mean $\langle E_r \rangle$

Now: Coupling between **DW, ZF**,
mean $\langle E_r \rangle$ and mean profile

Previous: Coupling between **DW**,
mean $\langle E_r \rangle$ and mean profile

Fluctuations

$$\frac{d\mathcal{E}}{dt} = \mathcal{E}\mathcal{N} - a_1\mathcal{E}^2 - a_2V^2\mathcal{E} - a_3V_{ZF}^2\mathcal{E}$$

Pressure gradient

$$\frac{d\mathcal{N}}{dt} = -c_1\mathcal{E}\mathcal{N} - c_2\mathcal{N} + Q$$

Zonal flow

$$\frac{dV_{ZF}}{dt} = b_1 \frac{V_{ZF}^2\mathcal{E}}{1 + b_2V^2} - b_3V_{ZF}$$

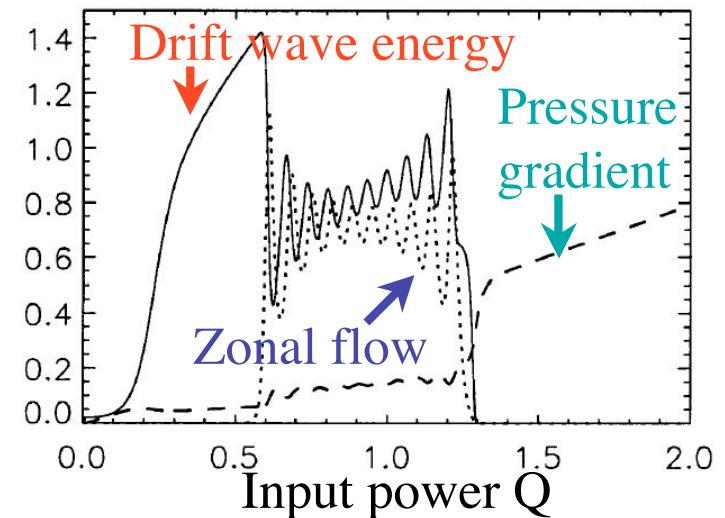
Mean flow

$$\frac{dV}{dt} = (V - d\mathcal{N}^2) + F_{\text{nonlinear}}$$

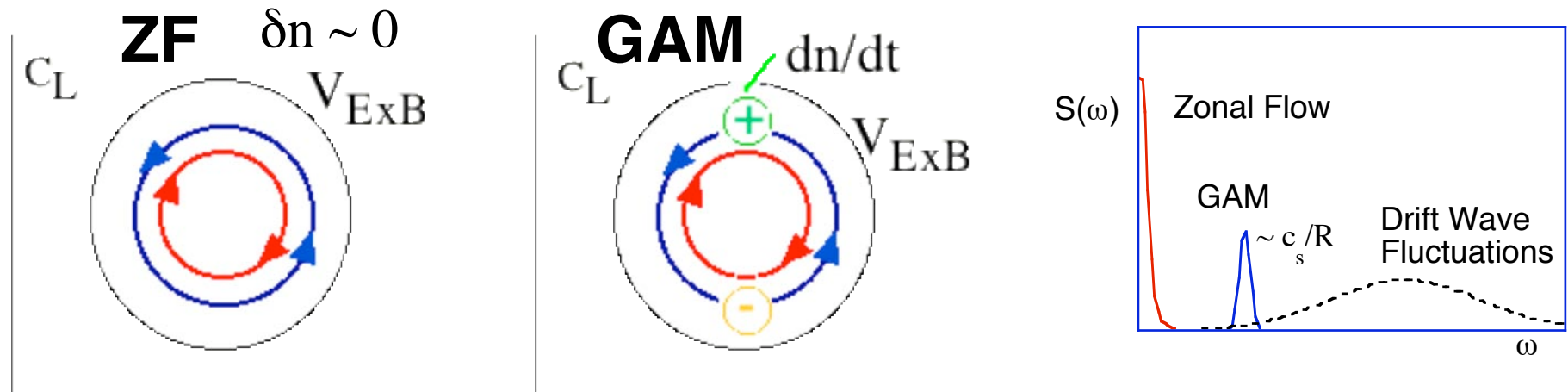
↑
Nonlinearity; orbit loss,
etc. in previous model

**New
coupling**

Prediction of
bifurcation, dither,
hysteresis, ...



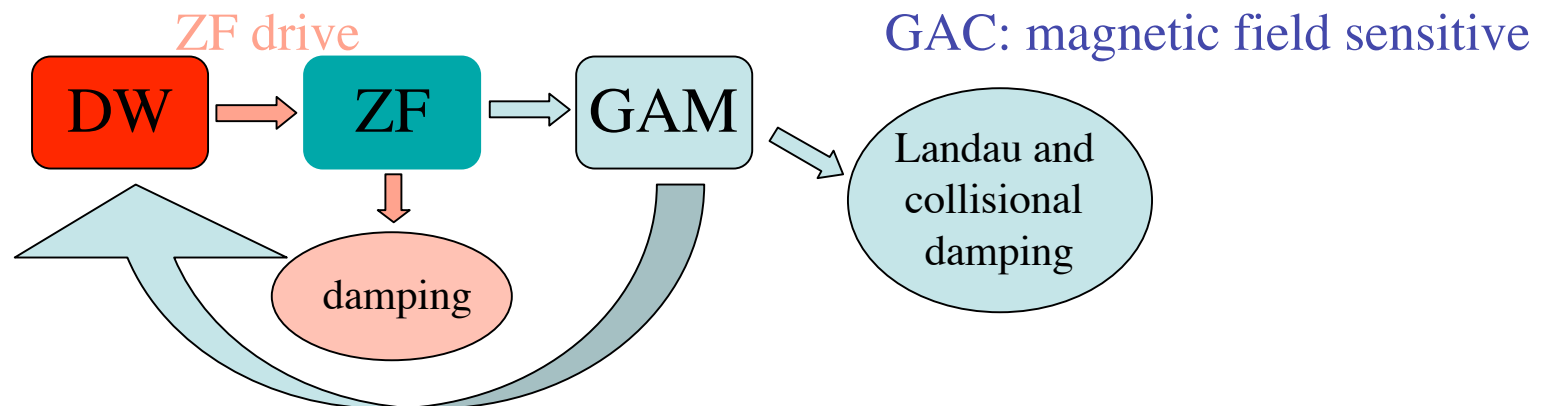
Zonal Flow and GAM: two kinds of secondary flow



GAMS: important near the edge Lower temperature: $v_{ii} \uparrow$, and $c_s/R \downarrow$

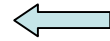
\Rightarrow ZF dynamics and GAM dynamics merge

New feature: geodesic acoustic coupling (GAC)



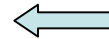
Control Knobs

(i) Zonal Flow Damping



Collisionality,
 ε , q , geometry, ...
n.b. especially stellarators

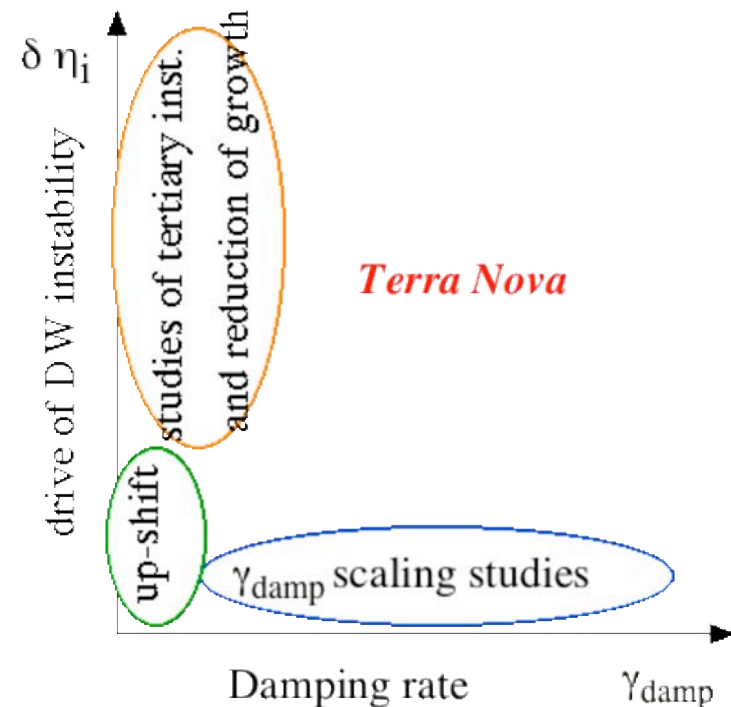
(ii) External Drive
(e.g., RF)



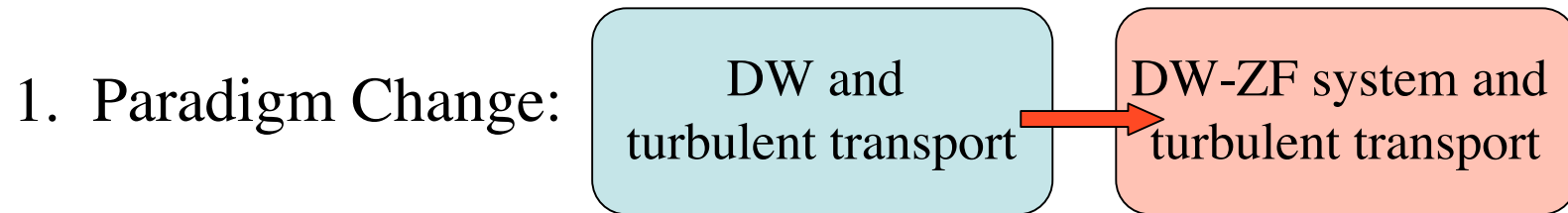
Choice of wave, Wave
polarity, launching, (e.g.,
studies of IBW), rational
surfaces...

III Future Research: "What we do not yet understand"

1. Experimentally convincing link between ZFs and confinement
2. Dominant collisionless saturation mechanism: selection rule ?
3. Quantitative predictability
 - (a) \mathcal{R} -factor ? - trends ??
 - (b) Suppression - γ_E vs γ_{Lin} ?
 - (c) How near marginality ?
 - (d) Effects on transition ?
 - (e) Flux PDF ?
4. Pattern formation competition
ZF vs. Streamers, Avalanches
5. Efficiency of control
6. Mini-max principle for self-consistent DW-ZF system ?



Summary



Linear and quasi-linear theory \longrightarrow Nonlinear theory
 (Simple way does not work.)

2. Critical for Fusion Devices

e. g., $\left\{ \begin{array}{l} \mathcal{R}\text{-factor} \xrightarrow{\text{Helps along}} \text{Route to ITER} \\ \text{Barrier Transitions} \end{array} \right.$

3. Progress and Convergence of Thinking on ZF Physics

4. Speculations: More Importance for Wider Issues

e.g., TAE, RWM, NTM; peak heat load problem, etc.

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P. H. Diamond: OV/2-1, This talk

Other member collaborators



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